

Methodology for Analysis of the Performance of Mesh-type Regenerators

Ignacio Carvajal^{*1}, Florencio Sanchez², Dario Omaña³, Samuel Miranda⁴

National Polytechnic Institute, Mechanical Engineering Department,

Av. IPN s/n Edif. 5, 3er piso, Col. Lindavista, 07738, Mexico City, Mexico

^{*1}icarvajal@ipn.mx; ²fsnchz@yahoo.com.mx; ³daomve@hotmail.com; ⁴chicote2.0@gmail.com

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Abstract

In this work, a methodology for analysis of the performance of wire meshes as regenerative media for Stirling engines is proposed. Methodology includes operation conditions of the engine and mesh physical characteristics, such as material and porosity, as parameters to predict the pressure drop and the thermal efficiency of the mesh. The proposed methodology was used to evaluate regenerators made of stainless steel wire meshes of 100, 150 and 250 threads per inch, of three different Stirling engines reported in the literature. In order to choose the best mesh to make a regenerator for a particular gamma type engine, the proposed methodology was applied to analyze the performance of meshes 100, 125, 250 and 500 threads per inch. After comparison it was found that mesh 250 presents higher thermal efficiency than meshes 100 and 125, and lower pressure drop than mesh 500. The main advantage of the proposed methodology is that it gives an overall performance of the regenerator in the early design stage.

Keywords

Stirling Engine; Regenerator; Pressure Drop; Thermal Efficiency; Wire Meshes

Nomenclature

A_{ff} = Free flow area, m²

A_{wg} = Heat transfer area, m²

d_w = Wire diameter, m

F = Design factor

L_{reg} = Characteristic length of regenerator

m_w = Threads per inch

n = Number of screens in the regenerator

p_m = Mean pressure, Pa

V_p = Piston swept volume, cm³

ϕ_v = Volumetric porosity

ω = Angular velocity, rad

τ = Temperature ratio, K

Introduction

One of the principal advantages of a Stirling cycle engine is that it can achieve a high efficiency by incorporating a regenerator. The regenerator acts as a temporal storage of energy; while the working fluid is traveling from the hot space, where it is expanded, to the cold space, where it is compressed, it passes through the regenerator cooling down while its heat is stored in the regenerator, once it begins to travel in the other direction from the cold space to the hot space the fluid recovers the heat stored from the previous cycle, heating up while it passes through the regenerator. This makes the engine more efficient as less energy is needed from the source to expand the working gas and less heat needs to be rejected in the heat sink. Thus, regenerators contribute to enhanced efficiency of the engine. For more heat transfer capacity, the regenerator needs to have a tight packed matrix inside, but this can yield to high pressure losses which impair the performance of the engine. Thus, the analysis of a regenerator needs to deal with its thermal efficiency and the pressure loss as well.

The importance of the study of the regenerator performance is reflected in the literature that has been developed in this matter. Organ (1994) has developed a method for calculating the amount of heat transferred on the regenerator and its inefficiency that accounts for pumping losses too, the method lies on the reappraisal of the assumptions made by Hausen and Nusselt for heat exchanging process analysis and

taking into account harmonic particle motion and low flush ratios. This model is a good solution for the regenerator evaluation problem and can be used to analyze the behavior of a specific design. However, application of this model could take a long time if it is used for comparison of many alternatives of regenerators when designing a Stirling engine.

Other studies have been made based on numerical analysis such as the one by Andersen, Carlsend and Thomsen (2006), however, the model proposed is not joined to the performance of a specific machine; or the work of Gary, Daney and Radebaugh (1984), in which the authors do not offer a full description of the algorithm so as to be reproduced and used in the analysis of a different design than the depicted.

In their work, Zhu and Matsubara (2004) presented a numerical model that could be used to analyze the performance of a regenerator for Stirling engine used in refrigeration. However, it requires validation for Stirling engines used in other applications, for example, for high temperature engines.

Tanaka et al. (1990) have built correlations for pressure drop in wire nets and sponge metals, and their results have been checked experimentally but not correlated with existing machines. On the other hand, Jones (1986) has developed an analytical method to evaluate the performance of the regenerator addressing its finite mass but it does not include the pressure losses.

In the works of Knowles (1997), Bin-Num and Manidakos (2004) and Takizawa (2002) there were proposed three new matrixes for regenerators and their methods for calculation. However, these methods cannot be used to calculate other type of matrixes for regenerators, such as conventional wire meshes, and some of them have complex fabrication processes.

From above mentioned, it can be seen that there is a lack of a simple analytical method to determine the amount of heat transferred within a Stirling engine regenerator and because of that, its thermal efficiency. The goal of this research is to make a simple methodology that involves the regenerator main parameters such as heat transfer rate, thermal efficiency and pressure drop. In addition, to make the proposed methodology more complete it should take in account the mesh physical characteristics, such as material and porosity.

The main advantage of the proposed methodology is that it can give an idea of the overall performance of the regenerator in the early design stage without

complex calculations.

The proposed methodology uses the correlations obtained by Gedeon and Wood (1996), based on their experimental observations, plus the use of non-dimensional numbers that are easy to handle and diminish considerably the number of variables involved in the calculations.

Material and Engines for Comparison

There are many materials in the market that can be used to make wire meshes; they vary in cost, availability and mainly in their physical properties.

Table 1 describes some of the available materials that can be used to make regenerators; in this table one can infer that the stainless steel is a good choice for regenerator fabrication. Its main characteristics are that it is less dense than copper, it has a low thermal conductivity, it can handle higher temperatures than the other materials compared and it is commercially available in the form of meshes of different gages. Also, it has to be noted that other authors used this material to fabricate their regenerators.

TABLE 1 PHYSICAL PROPERTIES OF DIFFERENT METALS

Material	ρ (kg/m ³)	c_p (kJ/kg K)	$k@100^\circ\text{C}$ (W/m K)	Melting point $^\circ\text{C}$
Stainless Steel 304	7817	0.460	15	1371
Aluminium	2707	0.896	206	659
Copper	8954	0.419	391	1086
Iron	7897	0.452	67	1535
Lead	11340	0.130	33.4	327

From a vast literature involving studies of regenerators, three references (Karabulut, 2000; Omaña, 2006; Organ, 1997) were chosen to build the correlations used in the proposed methodology. The selection was made because of the use of stainless steel regenerators and air, as working fluid, in the authors' experiments (Karabulut, 2000; Organ, 1997) and in the engine designed by Omaña, 2006 (see Fig. 1).

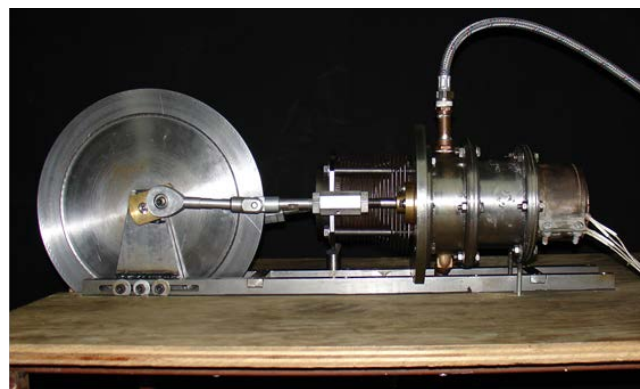


FIG. 1 GAMMA ENGINE 125-O (OMANA, 2006).

Tables 2 and 3 show the mesh characteristics and the technical specifications of the engines compared, respectively.

Methodology

For determining the amount of heat that is transferred within the regenerator, it is necessary to know the value of the heat transfer coefficient h , which can be obtained if one knows the value of the Nusselt number.

TABLE 2 MESHES USED FOR COMPARISON

Characteristic	Mesh 100	Mesh 125	Mesh 150	Mesh 250
Material	SS 304	SS 304	SS 304	SS 304
Wire diameter, mm	0.112	0.080	0.071	0.040
Volumetric porosity	0.70	0.69	0.67	0.69
Author	Karabulut	Omaña	Organ	Organ
Code	100-K	125-O	150-R	250-R

TABLE 3 ENGINE SPECIFICATIONS

Characteristic	100-K	125-O	150-R	250-R
Engine type	α	γ	β	β
Working fluid	Air	Air	Air	Air
Alpha angle	90°	90°	121°	121°
RPM max	850	—	1500	1500
Cooling fluid	Water	Water	Air	Air
Regenerator hydraulic diameter [mm]	30	15.5	34.04	34.04
Regenerator length (L_r) [mm]	24	60	28	28
Free flow area (A_{ff}) [m ²]	7.07 $\times 10^{-4}$	2.35 $\times 10^{-3}$	9.10 $\times 10^{-4}$	9.10 $\times 10^{-4}$
Heat Transfer Area (A_{wg}) [m ²]	0.212	2.167	0.473	0.788
Displacer swept volume [cc]	71.88	260.1	59.4	59.4
Piston swept volume [cc]	129.39	260.1	64.15	64.15
Maximum indicated power [W]	65	—	250	250
Test RPM	555	150	1500	1500
Test pressure [bar]	2.5	1	14	14
Expansion space temp [°C]	1100	600	700	700
Compression space temp [°C]	20	20	60	60

Equation 1 is presented by Gedeon and Wood (1996), for the calculation of the Nusselt number from the Peclet number. This correlation is valid for Reynolds numbers starting from 1.04 to 3400, this span is enough for the vast majority of Stirling engines to fit in.

$$Nu = (1 + 0.99Pe^{0.66})\phi_v^{1.79} \quad (1)$$

For the friction factor, Gedeon and Wood (1996) present the following correlation:

$$f = 0.5274 + \frac{68.556}{Re} \quad (2)$$

This correlation is valid for Reynolds numbers from 0.45 to 6100.

It is also necessary to calculate the power the engine

will deliver. Kontragool and Wongwises (2005) analyzed some mathematical expressions to calculate the power with a good approximation, the analysis yield the equation number 3. This expression doesn't include losses like mechanical losses.

$$P_{out} = p_m \cdot V_p \cdot F \cdot \omega \cdot \left[\left(\frac{1-\tau}{1+\tau} \right) \right] \quad (3)$$

The above equation works fine for gamma type engines in which the recommended design factor F is 0.35 (Kontragool and Wongwises, 2005).

With the equations presented a calculation method can be generated to obtain preliminary results about the performance of Stirling regenerators. For this methodology it's necessary to have the preliminary design characteristics of the engine, and then use this to calculate the heat transfer amount and pressure loss within the regenerator and then the power and efficiency of the Stirling engine.

For wire meshes the hydraulic radius is a function of the wire diameter and the porosity of the mesh, given by:

$$r_h = \frac{d_w}{4} \cdot \frac{\phi_v}{(1-\phi_v)} \quad (4)$$

Where the mesh porosity can be calculated by:

$$\phi_v = 1 - \frac{1}{4} \cdot \pi \cdot m_w \cdot d_w \quad (5)$$

The hydraulic radius is useful for determining the Reynolds number of the fluid crossing the regenerator, based on the mean air velocity:

$$Re = \frac{4\rho u r_h}{\mu} \quad (6)$$

Also, the mass flow rate and free flow area in the regenerator must be determined.

Once the Reynolds number is known we must calculate the Prandtl number defined as:

$$Pr = \frac{c_p \mu}{k} \quad (7)$$

And then the Peclet number:

$$Pe = Re \cdot Pr \quad (8)$$

With the Peclet number, the Nusselt number can be calculated using equation 1 and then the heat transfer coefficient can be calculated with the following equation:

$$h = \frac{Nu \cdot k}{L_{reg}} \quad (9)$$

The heat transfer rate within the regenerator can be now calculated using the heat transfer coefficient, the heat exchange area inside the regenerator and the maximum temperature differential between the two ends of the regenerator, if one assumes that the regenerator ends are at the same temperature that the expansion and compression spaces:

$$Q = -h \cdot A_{wg} \cdot (T_e - T_c) \quad (10)$$

This heat transfer rate divided by the period of each cycle gives the heat transferred in each cycle, which can be compared with the maximum heat that can be transferred making a regenerator 100% efficient given by the equation 11:

$$Q_{ideal} = m \cdot C_v \cdot (T_e - T_c) \quad (11)$$

The comparison between Q and Q_{ideal} gives the thermal efficiency, ε , of the regenerator for the given flow and heat transfer conditions.

For the pressure loss calculation we must determine the shape of the regenerator if it is a circular duct the friction factor is given by equation 12, which is related with the pressure loss in the duct by equation 13 (Bejan, 2004):

$$f_d = \frac{16}{Re} \quad (12)$$

$$\Delta p_d = f_d \frac{4L_{reg}}{D_h} \left(\frac{1}{2} \rho \cdot u^2 \right) \quad (13)$$

While the friction factor of the wire meshes is:

$$\Delta p = \frac{1}{2} f \cdot n \cdot \rho \cdot u^2 \quad (14)$$

So the total pressure loss has two components, one is the pressure loss given by the friction with the duct walls, Δp_d , and the other component is the pressure loss due to the friction with the regenerator's wire meshes, Δp .

$$\Delta p_t = \Delta p_d + \Delta p \quad (15)$$

Using the isothermal thermodynamic model of the Stirling cycle with imperfect regeneration with the efficiency of the regenerator calculated previously, the work per cycle can be calculated, and the mean pressure, p_m , determined if one knows the volume swept by the power piston. The indicated power of the engine is then calculated by equation 3.

Figure 2 shows the procedure to analyze a regenerator using the proposed methodology.

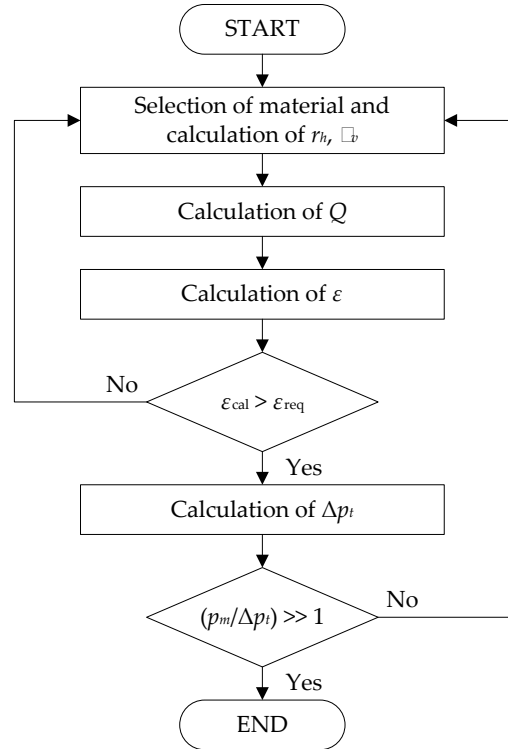


FIG. 2 DIAGRAM OF THE PROCEDURE TO ANALYZE A REGENERATOR USING THE PROPOSED METHODOLOGY

Results and Discussion

Table 4 shows the results obtained with the described method for the engines 100-K, 150-R and 250-R from which indicated power is already known according to the results previously published by other authors. The only data that can be used for comparison is the output power reported by a specific author, compared with the power calculated by the methodology.

The variation between the reported values and the calculated power efficiency is $\pm 20\%$ which is not acceptable; however, following the recommendations given by (Kontragool and Wongwises, 2005) a design factor is added to the equation 3. The factor of 0.35 used for gamma engines can be corrected for the other geometries alpha and beta to fit with the values of power reported previously. Then the factor for an alpha engine is 0.29 and for a beta engine is 0.43.

Following this procedure, the method was used to determine the best stainless steel mesh for the gamma engine 125-O. Figure 3 shows the efficiency comparison between the meshes number #100, #125, #250 and #500 when varying the temperature differential for the engine. As expected the mesh of #500 is capable of reaching the major efficiency value as it provides more heat transfer area than the other meshes.

TABLE 4 COMPARISON OF RESULTS REPORTED BY OTHER AUTHORS AND THE ONES CALCULATED BY THE PROPOSED METHODOLOGY

Characteristic	100-K		150-R		250-R	
	Reported	Calculated	Reported	Calculated	Reported	Calculated
Test RPM	555	555	1500	1500	1500	1500
Expansion space temp [°C]	1100	1100	700	700	700	700
Compression space temp [°C]	20	20	60	60	60	60
Mesh number	100	100	150	150	250	250
Pressure [kPa]	250	250	1400	1400	1400	1400
Heat transferred to the mesh per cycle [J]	—	−143.05	—	−59.63	—	−159.11
Maximum heat transfer allowable (Q_{ideal}) [J]	—	−821.91	—	−1715.63	—	−1718.42
Pressure loss in regenerator Δp_i (kPa)	—	17.40	—	77.24	—	205.72
Thermal efficiency of regenerator (ϵ)	—	0.17	—	0.03	—	0.09
Engine output power (P_{out}) [W]	65	79.15	250	203.24	250	203.24

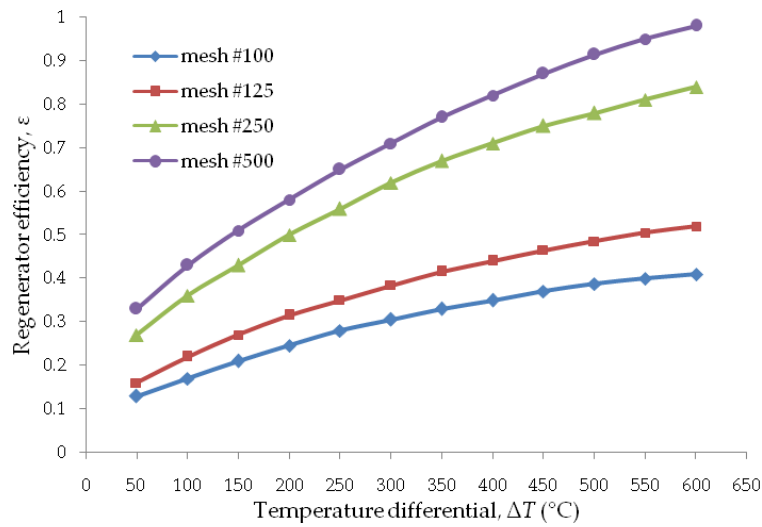


FIG. 3 THERMAL EFFICIENCY

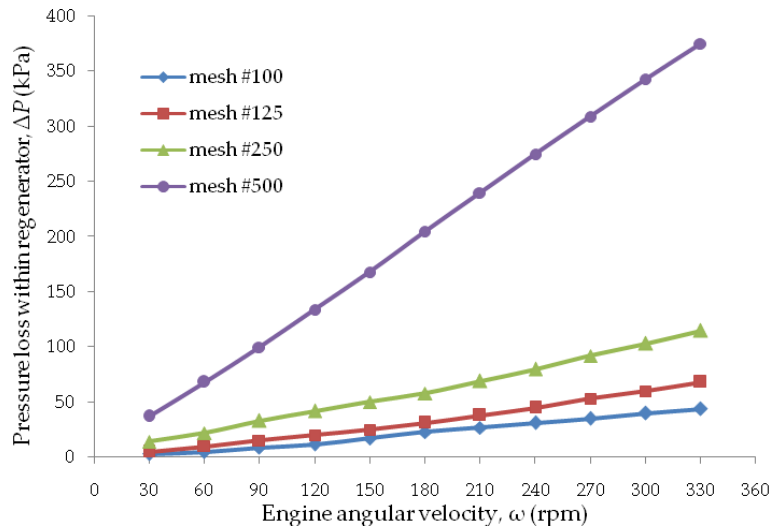


FIG. 4 PRESSURE DROP

However, it has to be noted that the shift between mesh #125 to #250, for a factor of 2, is not reflected between mesh #250 to #500. This can be explained using the relations of hydraulic radius and porosity. The value of the hydraulic radius for mesh #500 is approximately 60% lower than the one for the mesh

#250. For the same mean velocity of air, u , it means that the Re and Pe numbers for mesh #500 are lower in this proportion than the one for the mesh #250. On the other hand the porosity for the mesh #250 is higher approximately 20% than the one for the mesh #500. The influence of hydraulic radius and porosity is

reflected on the value of Nusselt number, calculated using equation 3, resulting that for mesh #500 it is lower than the one for the mesh #250. Therefore, mesh #500 reaches the major efficiency value because it provides more heat transfer area but mesh #250 has efficiency closer to it because it has a better heat transfer performance.

Figure 4 shows the total pressure drop values between the meshes compared when varying the angular velocity of the engine. The mesh of 500 threads per inch clearly surpasses the other meshes in pressure drop value; a mesh with this number of threads per inch offers a very intricate passage for the fluid flow.

In order to explain the quantum shift of pressure drop for mesh #500 when compared to mesh #250, the equations for the friction factor of wire meshes (2) and of duct (12) are used. As was stated above, the value of the hydraulic radius for mesh #500 is approximately 60% lower than the one for the mesh #250. For the same mean velocity of air, u , it means that the Re number and the values of friction factors for mesh #250 are higher in this proportion than the ones for the mesh #500. Moreover, because the wire diameter n (equation 14), of mesh #500 is half the diameter of mesh #250, the number of screens of mesh #500 packed in the regenerator is twice the number of screens of mesh #250. Therefore, the total pressure drop for mesh #500 is much higher compared to the one for mesh #250.

From the above mentioned, it can be seen that mesh #500 has the major thermal efficiency but the pressure drop when using this mesh can be an inconvenience as the pressure is directly related to the power output of the engine, if the relationship between mean pressure and pressure drop is small then the engine could stop suddenly or not work at all. When compared under this parameter the mesh #100 is the best option but its inconvenience is the low thermal efficiency it reaches when compared with meshes #250 or #500.

Best selection is the mesh #250 as it reaches a thermal efficiency only 13% under the mesh #500 while not causing a significant pressure loss as it is 3.24 times smaller than the pressure loss of #500 mesh.

The fabrication cost is also important when selecting a material for a regenerator. Table 5 was made with the information provided by a retailer of steel meshes in Mexico; it shows the cost of the material needed to make the regenerator for the gamma type Stirling engine 125-O.

TABLE 5 STAINLESS STEEL MESH COST

Mesh	Unitary cost (USD/m ²)	Area needed (m ²)	Total cost (USD)
#100	34.40	1.747	60.11
#125	40.00	2.167	86.67
#250	59.20	4.369	258.62
#500	160.00	8.737	1397.95

This table shows that cost of mesh #500 is too high to consider it as a viable option. At the same time, it can be seen that cost of mesh #250 is approximately 4 times when compared with mesh #100 and 3 times when compared with mesh #125. Thus, it is apparent that the best choice should be the meshes #100 or #125.

Figure 5 shows the influence of efficiency of regenerator, ε , on total efficiency of engine, E . It can be seen that an increment of efficiency of regenerator has greater effect on the total efficiency of engine when $\varepsilon > 0.7$. Therefore, meshes #100 or #125 do not fulfill this requirement (see Fig. 3). Also from figure 3 it can be seen that for a $\Delta T > 400$ °C mesh #250 presents an efficiency higher than 0.7, consequently it is the best choice even though its cost is higher.

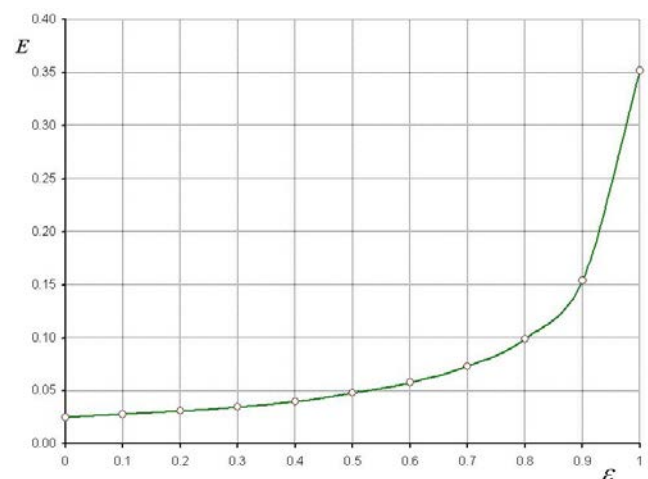


FIG. 5 EFFICIENCY OF REGENERATOR VERSUS EFFICIENCY OF ENGINE

Conclusions

When designing a Stirling engine, the regenerator performance is usually unknown. This methodology serves as a good guidance for compare the performance of different designs of regenerators using wire meshes, given only preliminary design characteristics of the engine such as swept volumes, desired engine speeds and operation temperature. The regenerator must be tailor made for a certain engine geometry and operation conditions, and then this methodology allows the designer to evaluate a regenerator design without making several lab

experiments or computer simulations. Once a design proposal exhibits good performance with this procedure it can be further investigated using numerical simulations.

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Ignacio Carvajal-Mariscal was born in Guadalajara, Mexico, on August 1, 1968. Mechanical Engineer, Moscow Power Engineering Institute, Moscow, Russia, 1994. M. S. and Ph. D. in Thermal Engineering, Moscow Power Engineering Institute, Moscow, Russia, 1996 and 1999 respectively. He is a Visiting Professor, Mechanical and Mechatronics Engineering Department, University of Waterloo, Waterloo, Canada, 2008-2009.

He is a full professor at the National Polytechnic Institute of Mexico since 2000 year. His research interests are heat transfer enhancement, experimental fluid dynamics and heat exchanger design.

Dr. Carvajal-Mariscal is an ASME member since 2008.



Florencio Sanchez-Silva was born in Mexico City, on October 27, 1952. Mechanical Engineer, National Polytechnic Institute of Mexico, Mexico City, 1974. M. S. in Thermodynamics and Ph. D. in Thermal Engineering, National Superior School of Mechanics and Aerothermics of Poitiers – France, 1980. He is a Postdoctorate in two-phase flow dynamics at the University of Pisa, Italy, 1988-1989. And he is Sabbatical stage at Texas A&M University, 2000-2001.

He is a full professor at the National Polytechnic Institute of Mexico since 1992. His research interests are two-phase flow dynamics, heat pipes and energy saving.

Dr. Sanchez-Silva is a fellow of the Société des Ingenieurset Scientifiques de France, Mexican Section (since 1987). He is a Member of the Mexican Academy of Sciences since 2004.